**MARINE PHYSICAL LABORATORY**

SCRIPPS INSTITUTION OF OCEANOGRAPHY

San Diego, California 92152

(5)

AD-A174 293

# Nonlinear Dynamics and Internal Waves

**FINAL REPORT**

Principal Investigator: H. D. I. Abarbanel
Final Report for Contract
N00014-83-K-0109
ARPA Order Number: 4696
For the Period: 1 January 1983 through 31 March 1985



DTIC FILE COPY



MPL-U-56/85



86 11 25 064

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

# REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: MPL-U-56/85
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): Final Report - Nonlinear Dynamics and Internal Waves
5. TYPE OF REPORT & PERIOD COVERED: Final Report
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): Henry D.I. Abarbanel
8. CONTRACT OR GRANT NUMBER(s): N00014-83-K-0109
9. PERFORMING ORGANIZATION NAME AND ADDRESS: University of California, San Diego, Marine Physical Laboratory of the Scripps Institution of Oceanography, San Diego, CA 92152
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: ARPA Order No. 4696
11. CONTROLLING OFFICE NAME AND ADDRESS: Defense Advanced Research Projects Agency 1400 Wilson Boulevard, Arlington Virginia 22209-2308
12. REPORT DATE: 1985
13. NUMBER OF PAGES: 4
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS. (of this report): Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the Defense Advanced Research Projects Agency or the U.S. Government.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

nonlinear dynamics
internal waves
transport coefficients
fluid and plasma flows

This research has concentrated on two topics:

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

During period of this contract research concentrated on two foci: (1) the evaluation of transport equations and transport coefficients relating to systems with widely differing spatial and temporal scales. This was motivated by the desire to understand the transfer of momentum, energy, etc. from the mesoscale to the smaller oceanic scales especially internal waves. (2) Nonlinear stability of fluid and plasma flows using this method developed by Arnol'd.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

A

# NONLINEAR DYNAMICS AND INTERNAL WAVES

Final Report
Advanced Research Projects Agency
Contract: N00014-83-K-0109
Period: 1 January 1983 through 31 March 1985
Principal Investigator: H.D.I. Abarbanel
Total Award: $120,000

During the period of this contract -- 1/3/83 to 3/31/85 my research took on two foci: (1) the evaluation of transport equations and transport coefficients relating to systems with widely differing spatial and temporal scales. This was motivated by the desire to understand the transfer of momentum, energy, etc from the mesoscale to the smaller oceanic scales especially internal waves. (2) Nonlinear stability of fluid and plasma flows using this method developed by Arnol'd.

In the first focus I studied with my students Keeler and Buchsbaum model systems where one is high frequency and the other is "slow" relative to it. Taken together the total system is Hamiltonian -- energy conserving and phase space volume preserving. The slow part (representing the mesoscale oceanic phenomean) will transfer energy and other physical properties to the more rapid system and thus "look" as if it is dissipative on time scales dictated by the phase space correlation functions of the fast system. That time scale measures the rate at which information which enters the fast

system gets lost in the sense of forgetting its initial conditions or origin. This work is still in progress but has led to the formulation of the mesoscale problem in Hamiltonian terms and to the understanding of several model systems created to imitate the setup just described. The most interesting of these model systems has the intriguing property of possessing an apparently infinite number of attractors with basins of attraction which are of zero volume (i.e. have dimension less than the ambient phase space). This has major implications (not yet fully explored) for the predictability of dynamical systems including those of central relevance for fluid flow.

In the second year, I began the study of nonlinear stability of certain fluid and plasma flows. The major application of this has been a paper published in Physical Review Letters in June, 1984 followed by a longer paper to appear in the Philosophical Transactions of the Royal Society. The main result there is that the classical stratified, shear flow configuration which is linearly unstable when the Richardson number is less than 1/4 is nonlinearly stable when that dimensionless number exceeds unity. This is a result more or less conjectured since the 1920's and complements the linear Miles-Howard result from 1961.

Publications emerging from the contract period:

(1) Bifurcation Phenomena in Collisional Plasmas; Symposium on Nonlinear

Problems in Engineering, Argonne National Laboratory, April 1983.

(2) Richardson Number Criterion for the Nonlinear Stability of Stratified Flow, with D. Holm, J. Marsden, and T. Ratiu; Phys. Rev. Letters, June 1984.

(3) Hamiltonian Formulation of Nearly Geostrophic Flow, to be published in Geophysical and Astrophysical Fluid Dynamics, Fall 1985.

(4) Nonlinear Stability of Stratified Fluids with D. Holm, J. Marsden, and T. Ratiu to appear in Phil. Trans. Roy. Soc., Fall 1985.

(5) Symmetries and the Interpretation of Nonlinear Stability in Three Dimensional Fluid Flow with D. Holm to be published in J. Fluid Mech.

(6) An Iterated Map with an Infinite Number of Attractors with S. Buchsbaum and J. Keeler, to be published in Physica D.

During and just after the contract period I delivered talks at the following places on work done under the auspices of the contract: UCLA, University of Hawaii, UCSD, UC Davis, Los Alamos National Laboratory, ONR-NSWC Conference on Nonlinear Dynamics in May, 1985, Woods Hole Oceanographic Institution, and UC Santa Barbara No written reports other than proceedings of the conference and the publications indicated above were issued.

# DISTRIBUTION LIST

Dr. A. Bruchheim
Defense Advanced Research Projects Agency
1400 Wilson Boulevard
Arlington, VA 22209

ONR Resident Representative
University of California, San Diego
La Jolla, CA 92093

Director
Naval Research Laboratory
Atten: Code 2627
Washington, D.C. 20375 (6)

Defense Technical Information Center
Bldg. 5
Cameron Station
Alexandria, VA 22314